Electromagnetic Theory as an Aid to Understanding Electromagnetic Field Analysis

Tomohiro KEISHI

Electromagnetic field analysis is an indispensable tool in the design and development of electrical devices and appliances. Software development has now made it possible to analyze electromagnetic fields with ease; however, in order to model problems and evaluate analysis results, it is still important to have a good understanding of electromagnetic theory. It is difficult for beginners to understand electromagnetics, but gaining such knowledge should help them to understand the workings of electrical devices and appliances.

Keywords: electromagnetics, electromagnetic field analysis, electric field analysis, magnetic field analysis

1. Introduction

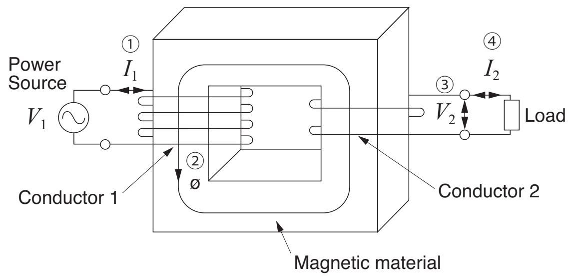
Advances in numerical analysis techniques and the advent of high-speed, high-capacity analytical hardware, such as the personal computer, have made it possible to perform electromagnetic field analysis using the finite element method and other numerical analysis techniques, thus making investigation of even the most complex electromagnetic phenomena possible. As such, electromagnetic field analysis has become an indispensable tool in the design and development of electrical devices and appliances(1).

As electromagnetic field analysis software has become increasingly powerful in recent years, analysis can now be performed very simply by just inputting the shape and setting conditions. When performing analysis, however, it is necessary to construct an electromagnetic field analysis model and evaluate analysis results appropriately. In order to do this, an understanding of electromagnetic phenomena based on electromagnetic theory is useful. In an ideal situation, such electromagnetic field analysis should be performed by those well versed in electromagnetics, but in an increasing number of cases such analysis is performed by those without any expertise. Furthermore, it would take a great deal of time and effort for novices in electromagnetics to learn from the beginning the fundamental principles of their discipline. This paper is intended to explain how knowledge of electromagnetics can lead to an understanding of the mechanisms of devices and appliances, so that those persons who are studying electromagnetics may develop an interest in this subject. At the end of this paper are brief descriptions of the bare essentials of what students of electromagnetics should know, and which the author hopes may serve as a reference.

2. Electromagnetic Phenomena of Electrical Devices and Appliances

By definition, electrical devices and appliances are devices that, through the use of electric currents *1 and magnetic fluxes *2, generate electric energy out of mechanical energy (generators), convert electric energy into mechanical energy (motors), or convert voltage to facilitate electric energy transmission (transformers). These are roughly divided into two types: transformers and rotary machines.

Figure 1 shows a typical example of a transformer. Conductor 1 is wound around a magnetic material and connected to a power source with voltage V_1 so that an electric current I_1 flows through the transformer's coil, generating magnetic flux ø. To concentrate the magnetic field that passes through it, Conductor 2 is wound around a magnetic material. If power-supply voltage V_1 is an alternating voltage, the voltage changes sinusoidally over time. Therefore, the magnetic flux also changes. This then generates electromotive force *3 V_2 in Conductor 2, and electric current I_2 flows if Conductor 2 is connected to a load (see Appendix 1 for details).



(1) Apply time-varying (alternating) electric current to Conductor 1
(2) Varying magnetic flux is generated and passes through the magnetic material and Conductor 2. (Conductor 2 and the magnetic flux are "interlinked.")
(3) Electromotive force is generated in Conductor 2.
(4) Electric current flows if Conductor 2 is connected to a load.

Fig. 1. Example of a Transformer

Conductors are used to carry electric current, while magnetic materials are used to encourage the magnetic flux to pass through. When the conductor path is altered, the electric current flows along it. Likewise, changing the magnetic material path can make the magnetic flux to flow along it.

The current sensor[1] shown in **Fig. 2** is also a type of transformer. The sensor's wire harness conductor corresponds to Conductor 1 of **Fig. 1**, but this sensor does not include Conductor 2, and a part of the magnetic material (magnetic core) is left open to create a gap.

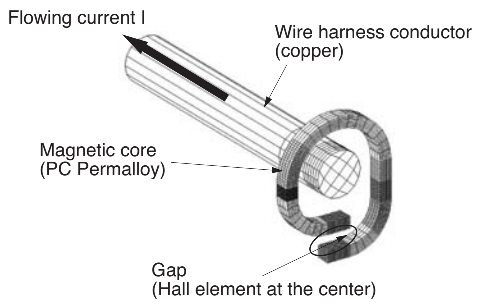


Fig. 2. Current sensor

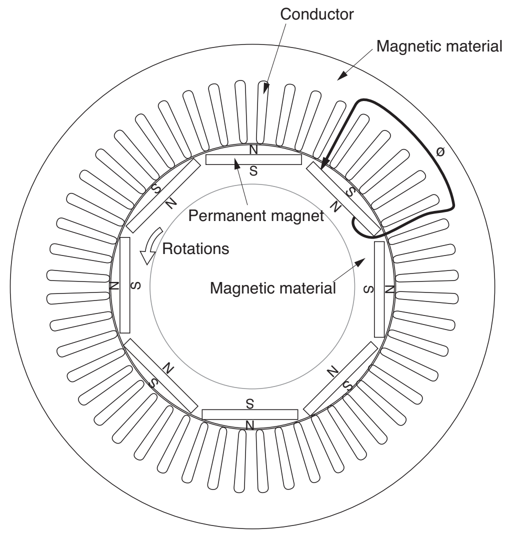


Fig. 4. Interior permanent magnet motor

Figure 3 is a representational model of a generator, an example of a rotary machine. Permanent magnets located above and below generate magnetic flux ø. A conductor is wound around a magnetic material, and they rotate together. Consequently, the magnetic flux passing through the conductor changes, thereby generating electromotive force V in the conductor. If the conductor is connected to a load, electric current I is applied (see Appendix 2 for details).

Figure 4 shows a cross section of an interior permanent magnet synchronous motor, which is an example of a rotary machine. Embedded inside the outer magnetic materials, conductors are organized into groups, to which three-phase alternating current is then applied. Permanent magnets are also embedded in the inner magnetic materials on the inside. The three-phase current applied to the outside conductors generates rotating magnetic fluxes in the magnetic materials and permanent magnets on the inside, which then produce turning force (torque) in the magnetic materials and permanent magnets on the inside (see Appendix 3 for details about motors).

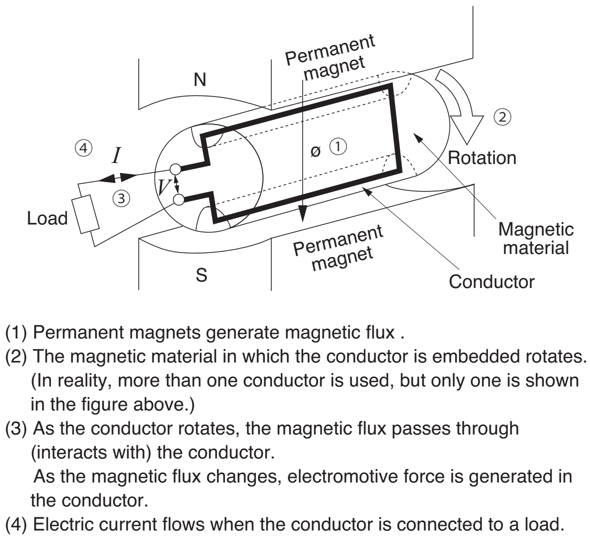


Fig. 3. Rotary machine

3. Basic Theories of Electromagnetics

3-1 Electric current *1

As shown in **Fig. 5**, electric current I[A] flows if direct voltage V[V] is applied at both ends of the conductor. The resistance R [Ω] of this conductor is given by **Equation (1)**, if the conductor has the same cross-section profile lengthwise and its cross section area, length, and resistivity are S[m^2], ℓ[m], and ρ[Ωm], respectively.

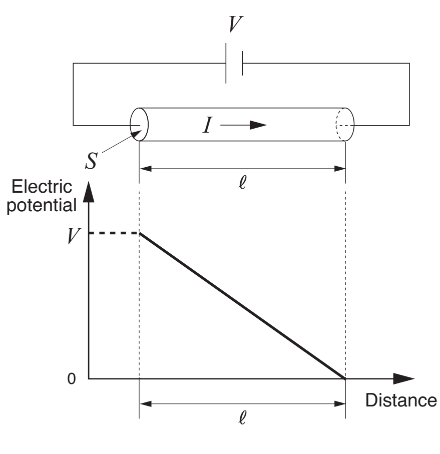


Fig. 5. Electric field by the current flowing through a conductor

$$R=\rho\frac{\ell}{S} \quad \cdots\cdots (1)$$

The relationship called Ohm's law, given by **Equation (2)**, exists among voltage V, electric current I, and resistance R.

$$V=IR \quad \cdots\cdots (2)$$

An electric field exists in the conductor, and an electric current flows through the conductor.

3-2 Electric field *4

In **Fig. 5**, due to the battery's voltage V, the electric potential at the left end of the conductor is V and that at the right end is 0. Since voltage V is applied over the conductor's entire length ℓ, electric field E[V/m] can be calculated using **Equation (3)**.

$$E=\frac{V}{\ell} \quad \cdots\cdots (3)$$

As shown in **Fig. 6**, the electric field generated when voltage V is applied to an insulator placed between parallel plate electrodes can also be obtained through **Equation (3)**.

Electric charge Q[C] that is accumulated in the electrodes is given by **Equation (4)**.

$$Q=CV \quad \cdots\cdots (4)$$

Here capacitance C[F] can be expressed by **Equation (5)**.

$$C=\varepsilon\frac{S}{\ell}=\varepsilon_r\varepsilon_0\frac{S}{\ell} \quad \cdots\cdots (5)$$

Where ε is the insulator's permittivity [F/m], ε_r is relative permittivity, and ε_0 is permittivity in a vacuum (8.854×10^{-12} F/m).

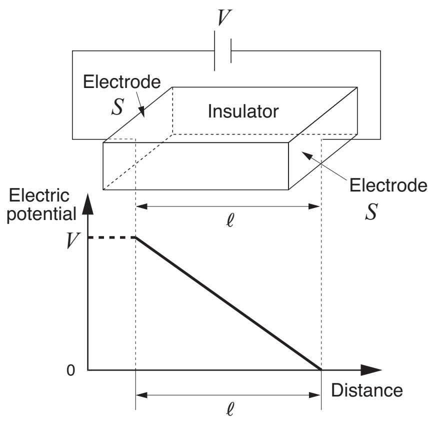


Fig. 6. Electric field of an insulator between parallel plate electrodes

Electric field E generated when voltage V is applied to an insulator placed between concentric cylindrical electrodes as shown in **Fig. 7** can be obtained through **Equation (6)** (see Appendix 4).

$$E=\frac{V}{r\ln\frac{r_2}{r_1}} \quad \cdots\cdots (6)$$

The capacitance of the insulator can be calculated by using **Equation (7)** (see Appendix 4).

$$C=\frac{2\pi\varepsilon}{\ln\frac{r_2}{r_1}} \quad \cdots\cdots (7)$$

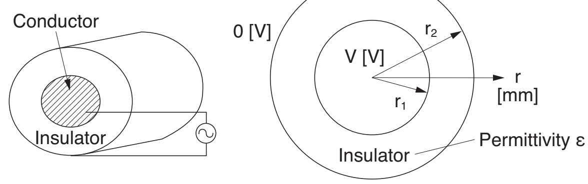


Fig. 7. Electric field of a concentric cylindrical electrode

3-3 Magnetic field *5

A magnetic field is produced around a conductor through which electric current I passes (**Fig. 8**). When a right-handed screw is rotated, a magnetic field is generated in the direction of the rotation, if an electric current is applied in the direction of the screw traveling forward (the right-hand rule). The strength of magnetic field H[A/m] at a radial distance r from the center of the conductor can be obtained by **Equation (8)** (see Appendix 5).

$$H=\frac{I}{2\pi r} \quad \cdots\cdots (8)$$

Magnetic flux density B[T] in the atmosphere is given by **Equation (9)**, if permeability in a vacuum is $\mu_0=4\pi\times10^{-7}$[H/m].

$$B=\mu_0H=\frac{\mu_0 I}{2\pi r} \quad \cdots\cdots (9)$$

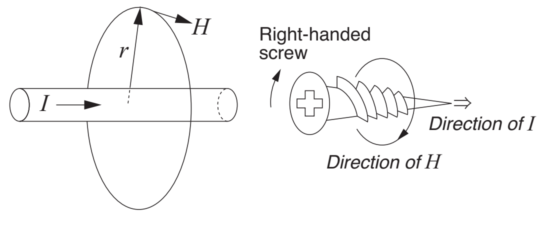


Fig. 8. Magnetic field created by electric current running through a conductor

As shown in **Fig. 9**, a magnetic field is produced around a permanent magnet.

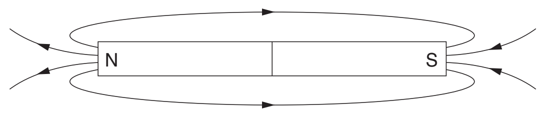


Fig. 9. Magnetic field created by a permanent magnet

3-4 Electromagnetic induction

When placing a conductor around another conductor through which time-varying electric current I passes and around which varying magnetic field H has been generated, electromotive force V_{out} is generated as shown in **Fig. 10**. If the conductor has a closed loop structure as shown in **Fig. 11**, electric current I_{out} flows.

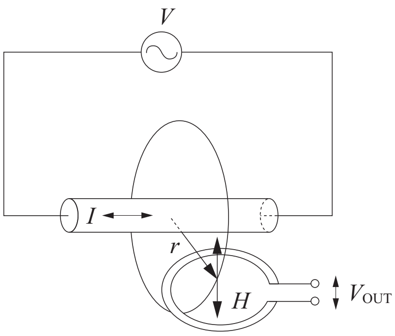


Fig. 10. Electromotive force generated by a varying magnetic field

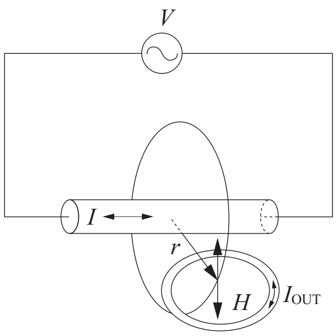


Fig. 11. Electric current generated by a varying magnetic field

As **Fig. 12** shows, when altering a magnetic flux that is linked with a conductor by moving another conductor within an area where magnetic field H exists, electromotive force V_{out} is generated. If the conductor has a closed loop structure as shown in **Fig. 13**, electric current I_{out} flows.

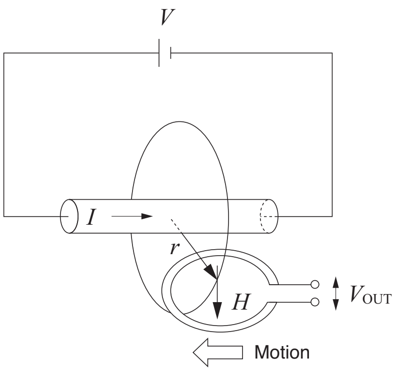


Fig. 12. Electromotive force generated by a varying magnetic field

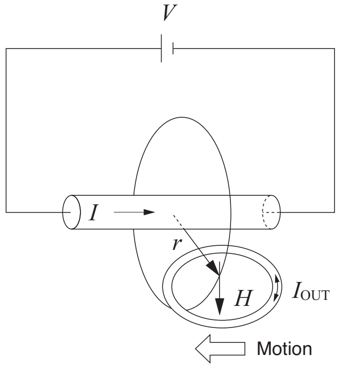


Fig. 13. Electric current generated by a varying magnetic field

4. Electromagnetic Consideration of Electromagnetic Field Analysis Results

4-1 Electric field

(1) Insulators between parallel plate electrodes

When voltage was applied to the insulator between parallel plate electrodes shown in **Fig. 14** and the electric field was analyzed using the finite element method (FEM), the electric field was found to be 5 [V/mm] ($E=5\times10^3$ [V/m]), which matched the theoretical value calculated using **Equation (3)**.

Based on the FEM electric field analysis results, capacitance is obtained as follows:

$$C=\frac{Q}{V}=\frac{\varepsilon ES}{V}$$
$$=\frac{1\times8.854\times10^{-12}\times5\times10^{3}\times1\times10^{-3}}{100}=4.43\times10^{-13}[\mathrm{F}]$$

Its theoretical value is calculated using **Equation (5)**:

$$C=\varepsilon\frac{S}{\ell}=\varepsilon_r\,\varepsilon_0\frac{S}{\ell}$$
$$=1\times8.854\times10^{-12}\times\frac{1\times10^{-3}}{20\times10^{-3}}=4.43\times10^{-13}[\mathrm{F}]$$

As demonstrated above, the FEM electric field analysis results matched the theoretical value.

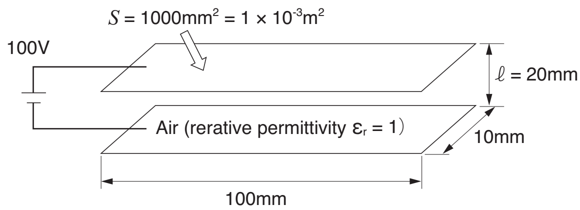


Fig. 14. Insulator betweem parallel plate electrodes

(2) Concentric cylindrical insulator

When voltage was applied to the concentric cylindrical insulator (XLPE cable insulator) shown in **Fig. 15** and the electric field was analyzed using FEM, the electric field matched the theoretical value calculated using **Equation (6)** as shown in **Fig. 16**.

Based on the FEM electric field analysis results, capacitance was calculated using electric field values of elements on the surface of the inner semiconductor layer:

$$C=\frac{Q}{V}=\frac{\varepsilon ES}{V}$$
$$=\frac{2.3\times8.854\times10^{-12}\times14.46\times10^{6}\times2\pi\times33.1\times10^{-3}\times1}{288.675\times10^{3}}$$
$$=2.12\times10^{-10}[\mathrm{F/m}]$$

Based on **Equation (7)**, the theoretical value is obtained as follows:

$$C=\frac{2\pi\varepsilon}{\ln\frac{r_2}{r_1}}=\frac{2\pi\varepsilon_r\varepsilon_0}{\ln\frac{r_2}{r_1}}$$
$$=\frac{2\pi\times2.3\times8.854\times10^{-12}}{\ln\frac{60.1}{33.1}}=2.15\times10^{-10}[\mathrm{F/m}]$$

The FEM electric field analysis results matched the theoretical value (1.4% margin of error).

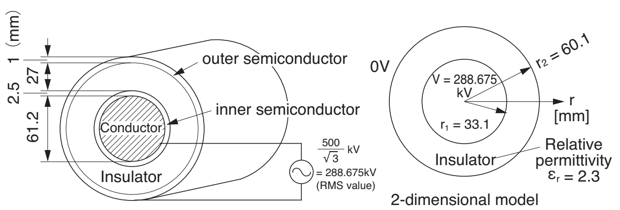


Fig. 15. Concentric cylindrical insulator

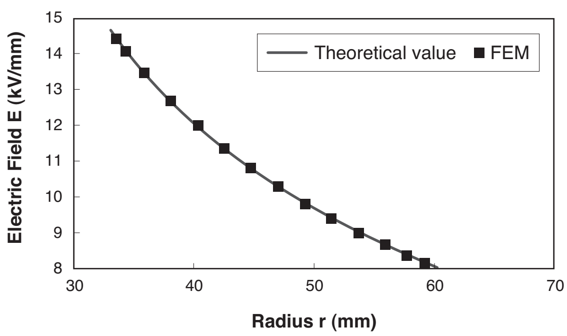


Fig. 16. Electric field of a concentric cylindrical insulator

4-2 Magnetic field

When electric current was passed through the linear circular conductor shown in **Fig. 17** and the magnetic field was analyzed using FEM, magnetic flux density was calculated as shown in **Fig. 18**. The results obtained with FEM analysis matched the theoretical value calculated using **Equation (9)**.

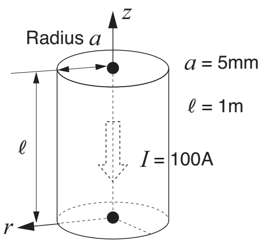


Fig. 17. Linear circular conductor

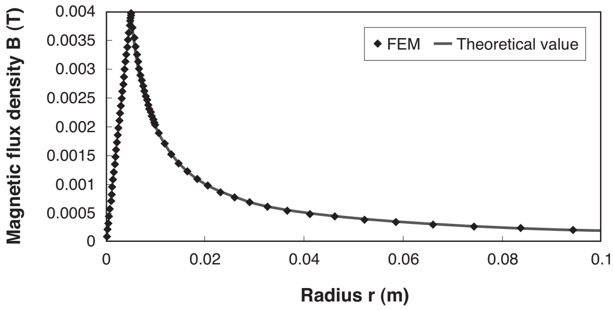


Fig. 18. Magnetic field around a linear circular conductor

5. Conclusion

Electrical devices and appliances achieve their intended functions through the interaction of an electric current and a magnetic flux. Electric current passes through the conductor, and a magnetic flux runs through the magnetic material. Electric current that passes through the conductor forms a magnetic flux, which in turn runs through the magnetic circuit made of magnetic materials. When the magnetic flux that runs through the inside of the looped conductor is altered, electromotive force is generated in the conductor. Electromotive force causes the electric current to flow, and electric current flows once a conductor is connected to a load, etc. to form a closed circuit. Permanent magnets can also be used to create a magnetic flux.

Electromagnetic theory explains such electromagnetic phenomena. Since physical quantities such as electric fields and magnetic fields are vectors, equations using vector analysis are employed, and these have been compiled into Maxwell's Equations(1). These equations can be resolved using numerical tools such as FEM, where a set of conditions are given, including the shapes of the objects to be analyzed, permittivity, permeability, and other physical properties, boundary conditions, and flowing electric current.

It was confirmed that electromagnetic field analysis results matched the theoretical values of electromagnetics in simple problems on electric fields and magnetic fields.

When designing electrical devices and appliances, this numerical solution – electromagnetic field analysis – is used. Knowledge of electromagnetics plays an important role in the development of analysis models and assessing analysis results.

The author hopes that the above explanations will help the reader to develop an interest in and understanding of electromagnetics.

Technical Term

*1 Electric current : The flow of electric charge carried by moving electrons and other charged particles. It is defined as the rate at which an electric charge travels through a given surface per unit of time, and is measured in amperes [A]. For an electric current of 1 ampere, 1 coulomb [C] per second of electric charge will pass through any plane (1 A = 1 C/s).
Electric charge: A form of charge found on elementary particles of matter. Electric charge can be either positive or negative and is measured in coulombs [C].

*2 Magnetic flux : Magnetic flux flows along the magnetic line of force from the north to south poles of permanent magnets. It is the sum of the products of normal direction components multiplied by the cross-sectional area of the density of the magnetic flux that passes through a given section of a magnetic field. Magnetic flux is measured in [Wb].
Magnetic flux density: Magnetic flux divided by the area through which it passes. Magnetic flux density is measured in [T]. ([Wb/m^2] = [T])
$\phi = BS$
Where ø is magnetic flux [Wb], B is magnetic flux density [T], and S is the cross-sectional area through which the magnetic flux passes [m^2]

*3 Electromotive force : A force that pushes charged particles such as electrons, thus causing a current to flow, equivalent to the potential difference (voltage) that gives rise to current. Electromotive force is measured in [V].

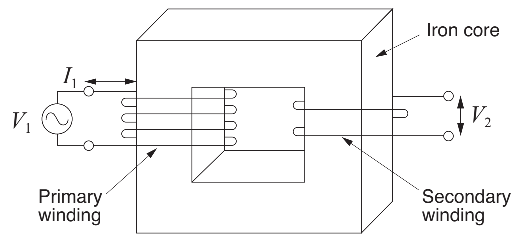
*4 Electric field : A region of space in which force is exerted on electrically charged objects. Electric fields are measured in [V/m].

*5 Magnetic field : When a permanent magnet is placed on a piece of paper over which iron sand has been sprinkled, the iron sand will form a regular pattern of lines around the magnet. This occurs in response to the magnetic field of the magnet. In addition to permanent magnets, a magnetic field also occurs around electric current. Magnetic fields are expressed through their magnetic flux density [T] or intensity [A/m].
$B = \mu H$
Where B is magnetic flux density [T], H is the magnetic field [A/m], and μ is permeability [H/m].

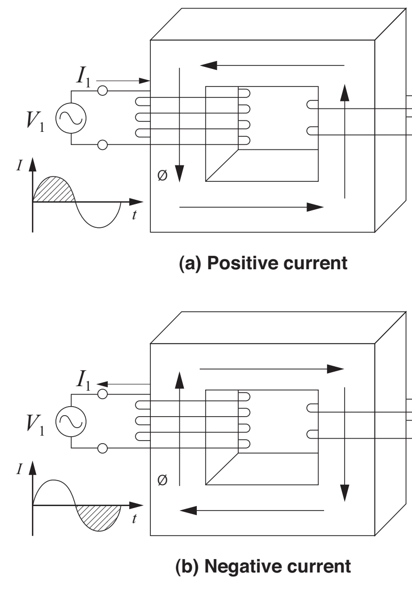
*6 Magnetic circuit : A path through which a magnetic flux that is made up of magnetic materials, etc. travels. Because Ohm's law applies to a magnetic circuit, where magnetic flux corresponds to electric current, magnetomotive force to voltage, and magnetic resistance to electric resistance, a magnetic circuit can be calculated in the same manner as that for electric circuits.
Magnetomotive force: Force that produces magnetic flux in a magnetic circuit. It is the product of electric current that flows in a coil multiplied by the winding number (number of turns) of the coils that are wound around an iron core.

Appendix

1. Transformers
As shown in **Appended Fig. 1**, in a transformer, a primary winding conductor and a secondary winding conductor are wound around the iron core, thereby forming a closed magnetic circuit*6.
As shown in **Appended Fig. 2**, when alternating current is applied to the primary winding, a time-varying magnetic flux is produced and flows through the iron core, which then induces a time-varying magnetic flux in the secondary winding's iron core.



Appended Fig. 1. Electromotive force generated by a transformer

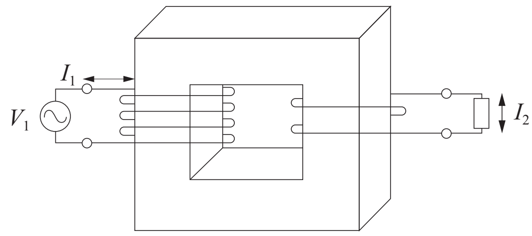


Appended Fig. 2. Magnetic flux generated by a transformer

As the magnetic flux inside the secondary winding changes, electromotive force is produced in the conductor. If the voltage of the primary winding is V_1[V], the winding number (number of turns) of the primary winding is N_1, and that of the secondary winding is N_2, electromotive force V_2[V] is:

$$V_2 = \frac{N_2}{N_1} V_1 \quad \text{(App. 1)}$$

When a closed circuit is formed by having a load connected to the secondary winding or by other means, electric current flows as shown in **Appended Fig. 3**.

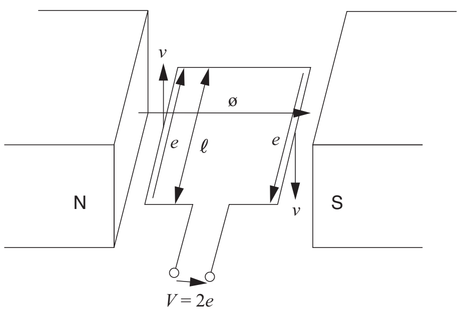


Appended Fig. 3. Electric current generated by a transformer

2. Generators
As shown in **Appended Fig. 4**, by rotating the conductor in the magnetic flux ø[Wb] at a speed of v[m/s] to cut the magnetic flux, electromotive force e[V] is generated over the length ℓ[m] where the conductor cuts across the magnetic flux. Electromotive force e is generated both in the magnetic flux and the conductor that is perpendicular to it. Since these two forces are facing in opposite directions, electromotive force $V = 2e$[V] is generated at the ends of the conductor. This is how a generator functions.

$$e = vB\ell \quad \text{(App. 2)}$$

If the area through which magnetic flux passes is S[m^2], magnetic flux density B[T] is given by Equation (App. 3):



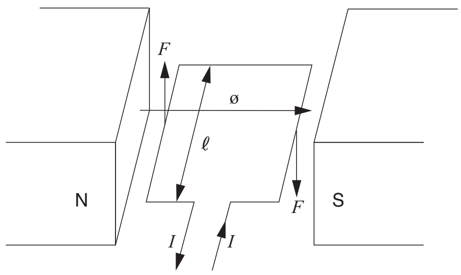
Appended Fig. 4. Principle of generators

$$B = \frac{ø}{S} \quad \text{(App. 3)}$$

3. Motors
As shown in **Appended Fig. 5**, by applying electric current I[A] to a conductor in magnetic flux ø[Wb], length ℓ[m] where the conductor cuts across the magnetic flux is subjected to force F[N]. Both the magnetic flux and the conductor that is perpendicular to it are subjected to force F, and since these two forces are facing in opposite directions, the conductor is subjected to turning force (torque). This is how a motor functions.

$$F = IB\ell \quad \text{(App. 4)}$$

If the area through which magnetic flux passes is S[m^2], magnetic flux density B[T] is given by Equation (App. 3).
In an actual motor, magnetic flux is generated and electric current is applied in such a way that force is always generated in the same direction as the rotation, even as the conductor itself rotates.



Appended Fig. 5. Principle of motors

4. Electric field of concentric cylindrical insulators
An electric field of an insulator placed between concentric cylindrical electrodes is calculated as shown in **Appended Fig. 6**.
From Gauss's law,

$$\int_S \mathbf{E} \cdot \mathbf{n} \, dS = \frac{Q}{\varepsilon} \quad \text{(App. 5)}$$

Where **E** is the electric field [V/m], **n** is the unit normal vector of a surface, S is area[m^2], Q is the electrode's electric charge [C], and ε is permittivity of the insulator [F/m], and the integral is the surface integral of the planes surrounding the electric charge.
If E is the electric field at the outer position of radius r[m] and the electric charge per every 1 m of unit length is Q, then from the following equation,

$$2\pi r \times 1 \times E = \frac{Q}{\varepsilon}$$

$$E = \frac{Q}{2\pi\varepsilon} \frac{1}{r} \quad \text{(App. 6)}$$

Applied voltage V is an integral of the electric field in Equation (App. 6).

$$V=\int_{r_1}^{r_2} E\mathrm{d}r=\int_{r_1}^{r_2}\frac{Q}{2\pi\varepsilon}\frac{1}{r}dr=\frac{Q}{2\pi\varepsilon}[\ln r]_{r_1}^{r_2}=\frac{Q}{2\pi\varepsilon}\ln\frac{r_2}{r_1}$$

Thus

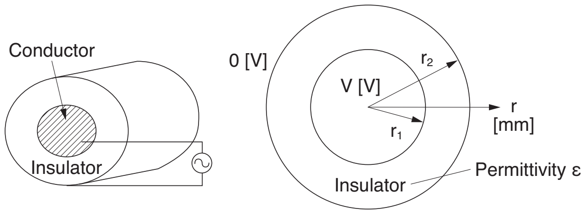
$$\frac{Q}{2\pi\varepsilon}=\frac{V}{\ln\frac{r_2}{r_1}}$$

When substituting this into Equation (App. 6),

$$E=\frac{V}{r\ln\frac{r_2}{r_1}} \quad \text{..........(App. 7)}$$

Capacitance C[F] is then calculated as follows.

$$C=\frac{Q}{V}=\frac{2\pi\varepsilon}{\ln\frac{r_2}{r_1}} \quad \text{..........(App. 8)}$$



Appended Fig. 6. Electric field of a concentric cylindrical insulator

5. Magnetic field around a linear current
A magnetic field around a conductor through which electric current flows is calculated as shown in **Appended Fig. 7**.
From Ampere's circuital integral law,

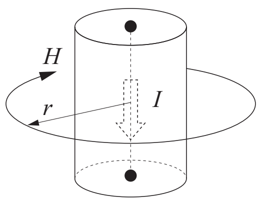
$$\oint \mathbf{H}\cdot\mathrm{d}\mathbf{s}=I \quad \text{..........(App. 9)}$$

Where **H** is the magnetic field [A/m], **s** is the linear vector, and I is the conductor's electric current [A], and the integral is the curvilinear integral on the contour surrounding the electric current.
From Equation (App. 9), $2\pi rH=I$,

$$H=\frac{I}{2\pi r} \quad \text{..........(App. 10)}$$

If permittivity in a vacuum is μ_0, the magnetic flux density $\boldsymbol{B}$[T] is

$$B=\frac{\mu_0 I}{2\pi r} \quad \text{..........(App. 11)}$$



Appended Fig. 7. Magnetic field around a linear cylindrical conductor

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(3) Naohei Yamada, "Electromagnetics," The Institute of the Electrical Engineers of Japan (IEEJ), 1971
(Reference (2) and (3) are written in Japanese.)

Contributor

T. KEISHI

• Senior Specialist
Dr. Engineering, Professional Engineer Japan (Electrics & Electronics Engineering)
Senior Assistant General Manager, Analysis Technology Research Center
Engaged in research of computer aided engineering (CAE)